UNSTEADY WAVE FLOWS OF A GAS SUSPENSION WITH ACCOUNT FOR PHASE CHANGES

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A system of equations of unsteady motion of a gas suspension with account for heat and mass transfer is presented. As a result of the computational experiment, the effect of the gas vibrations at the channel inlet on the motion of the gas suspension and the rate of heat and mass transfer is studied. Specifically, it is shown that, in the considered parametric region, the gas vibrations enhance the interphase heat and mass transfer.

An unsteady (pulsating, pulsed) motion of a gas (liquid) enhances heat and mass transfer in a number of technological processes, specifically, drying, diffusion, dissolution, etc. Gas pulsations at certain parameters increase the relative velocity of phases and decrease the thickness of the diffusion boundary layer. This also takes place under the conditions of pneumatic transport, which is widely used for drying dispersed materials. Gas pulsations can be generated, for example, by pulsating combustion chambers, valves, vibration emitters, etc.

This work studies the wave motion of a gas suspension in a pneumatic channel with account for heat and mass transfer. Consideration is given to the motion of a two-phase dispersed mixture of particles with a carrying phase (gas), which consists of two components, namely, a dry air (combustion products) and a water vapor. The system of the equations of motion is formulated using the equations of the gasdynamics of inter-penetrating continua of compressible media. For them to be applicable, it is assumed that the dimensions of the particles are markedly smaller than their minimum spacing, and the latter is much smaller than the distance at which kinematic and dynamic characteristics of the flow vary noticeably. This basic assumption in this case is regarded as feasible. Moreover, the following assumptions, which simplify a mathematical description, are adopted. The mixture is monodispersed, with the volume concentration of the dispersed phase being not too high, $\varepsilon_2 \ll 1$, owing to which particle interaction and collision can be neglected. The breakdown, coagulation, and formation of new dispersed particles are absent. The heat transfer and friction at the channel walls are also disregarded. The gas viscosity is taken into account only when the carrying phase interacts with the particles, since the Reynolds number that characterizes the gas flow is large.

The differential equations of conservation of the phase masses, momenta, and energy have the following form [1].

The equations of conservation of the phase masses are of the form

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial (\rho_1 v_1)}{\partial x} = n j_{21} , \tag{1}$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial (\rho_2 v_2)}{\partial x} = n j_{12} ; \tag{2}$$

The equation of conservation of the number of dispersed particles is

$$\frac{\partial n}{\partial t} + \frac{\partial (nv_2)}{\partial x} = 0 ,$$ (3)

with the reduced phase densities ρ_1 and ρ_2, which characterize the phase masses in unit volume of the mixture, being

$$\rho_1 = \varepsilon_1 \rho_1^0 , \quad \varepsilon_1 + \varepsilon_2 = 1 , \quad \varepsilon_2 = \pi d^3 n/6 , \quad \rho_2^0 = \rho_2/\varepsilon_2 , \quad j_{21} = -j_{12} ;$$

the equation of conservation for a dry air is

$$\frac{\partial \rho_3}{\partial t} + \frac{\partial (\rho_3 v_1)}{\partial x} = 0 ,$$ (4)

$$\rho_3^0 = \rho_3/\varepsilon_1 , \quad \rho_4^0 = \rho_1^0 - \rho_3^0 .$$

The equations of momenta of the phases are of the form

$$\frac{\partial (\rho_1 v_1)}{\partial t} + \frac{\partial (\rho_1 v_1^2)}{\partial x} = -\left(1 - \frac{3}{2}\varepsilon_2\right)\frac{\partial P}{\partial x} + F_1 ,$$ (5)

$$\frac{\partial (\rho_2 v_2)}{\partial t} + \frac{\partial (\rho_2 v_2^2)}{\partial x} = -\frac{3}{2}\varepsilon_2 \frac{\partial P}{\partial x} + F_2 ,$$ (6)

where

$$F_1 = -\left(1 - \frac{3}{2}\varepsilon_2\right)nf - \frac{3}{2}\varepsilon_2 \, nj_{12}v_{12} - nj_{12}v_2 ,$$

$$F_2 = \left(1 - \frac{3}{2}\varepsilon_2\right)nf + \frac{3}{2}\varepsilon_2 \, nj_{12}v_{12} + nj_{12}v_2 - \rho_2 \, g_2 ,$$

$$v_{12} = v_1 - v_2 .$$

The equation of the total energy of the mixture is of the form

$$\frac{\partial (\rho_1 E_1 + \rho_2 E_2)}{\partial t} + \frac{\partial}{\partial x}[\rho_1 v_1 E_1 + \rho_2 v_2 E_2 + P(\varepsilon_1 v_1 + \varepsilon_2 v_2)] = \rho_2 \, g_2 v_2 ,$$ (7)

and the equation of the influx of the heat of the dispersed phase is of the form

$$\frac{\partial (\rho_2 e_2)}{\partial t} + \frac{\partial (\rho_2 e_2 v_2)}{\partial x} = Q_2 .$$ (8)

The total and internal energy of the phases, the heat influx to the dispersed phase, and the equation of state appear as

$$E_i = e_i + \frac{v_i^2}{2} , \quad e_1 = c_1 T_1 , \quad e_2 = c_2 T_2 , \quad Q_2 = nq_2 , \quad P = \rho_1^0 R^* T_1/M_1 .$$ (9)

The saturation temperature as a function of the pressure over a fairly wide range is determined from the equation [1]

$$T_s(P) = T'/\ln(P'/P) ,$$

where T' and P' are the approximation parameters; $T' = 4996$ K and $P' = 6.4072 \cdot 10^{10}$ Pa.

The intensity of the phase change in the direction $2 \to 1$ per dispersed particle is expressed as

$$j_{21} = 4\pi R^2 D \, \text{Sh} \, \frac{\rho_{4s}^0 - \rho_4^0}{d} , \tag{10}$$

where $\rho_{4s}^0 = P_s M_4 / R^* T_s$.

The interphase heat and mass transfer is defined in terms of the coefficients of heat and mass transfer (or dimensionless Nusselt and Sherwood numbers) with the aid of experimental relations. In this case, where the amplitude of displacement of the medium A is much in excess of the particle diameter the flow over the particles can be considered as quasisteady, i.e., the velocity field at each instant of time obeys the laws of steady flow. The displacement amplitude is related to the amplitude of the vibrational velocity of the gas and frequency as $A = v_1^A/(2\pi\omega)$. At $v_1^A = 30$ m/sec and $\omega = 100$ Hz, the displacement amplitude is $A = 0.05$ m; therefore, $A/d_2 \gg 1$. This condition is generally always fulfilled in the plants, which operate with a low vibration frequency. Here, the heat and mass transfer can be assumed to be quasisteady [2, 3]. Then, the coefficient of mass transfer can be determined from the equation [1]

$$\text{Sh} = 2 + 0.55 \, \text{Re}_{12}^{0.5} \, \text{Sc}^{0.33} , \tag{11}$$

$$\text{Re}_{12} = \frac{d \, |v_1 - v_2|}{\nu_1} , \quad \nu_1 = \mu_1/\rho_1 .$$

The condition of phase equilibrium at the interface $T_{2R} = T_s(P)$ is assumed. Here, it is possible to neglect the thermal resistance inside the particles, setting $T_2 = T_{2R}$. The equation of the heat influx at the interface is of the form

$$q_2 = j_{12} r + q_1 , \tag{12}$$

$$q_1 = -4\pi R^2 \lambda_1 \, \text{Nu} \, \frac{T_{2R} - T_1}{d} , \tag{13}$$

where $\text{Nu} = 2 + 0.55 \, \text{Re}_{12}^{0.5} \, \text{Pr}^{0.33}$.

The relation for the force of interphase friction, acting on a particle, has the form

$$f = f^0 \pi R^2 \frac{\rho_1^0 \, |v_1 - v_2| \, (v_1 - v_2)}{2} ,$$

$$f^0 = f^1 \psi_\epsilon \, \psi , \quad f^1 = \frac{24}{\text{Re}_{12}} + \frac{4}{\sqrt{\text{Re}_{12}}} + 0.4 ,$$

with corrections for the flow confinement

$$\psi_\epsilon = (1 - \epsilon_2)^{-2.7}$$

and the compressibility of the carrying phase

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$$\psi = 1 + \exp\left(-\frac{0.427}{M_{12}^{4.63}} - \frac{3}{Re_{12}^{0.88}}\right).$$

Here, the Mach number is

$$M_{12} = \left(\frac{M_1}{\gamma R^* T_1}\right)^{0.5} |v_1 - v_2|.$$

The thermal conductivity of the gas and the dynamic viscosity are determined from the Sutherland equation:

$$\frac{\lambda_1}{\lambda_0} = \frac{\mu_1}{\mu_0} = \frac{273 + C}{T_1 + C}\left(\frac{T_1}{273}\right)^{3/2},$$

where C is the Sutherland constant, which is $C = 124$ for the air; $\mu_0 = 17.3 \cdot 10^{-6}$ Pa·sec and $\lambda_0 = 0.0244$ W/(m·K).

The diffusion coefficient of the vapor is

$$D = D^*\left(\frac{P^*}{P}\right)\left(\frac{T_1}{273}\right)^{3/2},$$

where $P^* = 101,300$ Pa and $D^* = 22 \cdot 10^{-6}$ m/sec.

The specific heat of the solid phase is

$$c_2 = c_2^0(1 - W_2) + c_5 W_2;$$

here, $W_2 = U_2/(U_2 + 1)$.

The moisture content of the particles is

$$U_2 = \frac{\rho_2^0}{\rho_6} - 1.$$

The specific heat of the gas (the carrying phase) is

$$c_1 = \frac{1}{U_1 + 1}(c_3 + c_4 U_1), \quad U_1 = \rho_4^0/\rho_3^0.$$

The initial conditions are

$$t = 0: \quad v_1 = \bar{v}_1; \quad P = P_0; \quad T_1 = 473 \text{ K};$$

$$\rho_1^0 = P_0 M_1/(R^* T_1); \quad v_2 = 0; \quad T_2 = 293 \text{ K};$$

$$\rho_2^0 = 900 \text{ kg/m}^3; \quad \varepsilon_2 = \varepsilon_{20}; \quad \rho_3 = \rho_1^0 \varepsilon_1/(U_{10} + 1).$$

The boundary conditions were specified as follows:

$$X = 0: \quad v_1 = \bar{v}_1 + v_1^4 \sin(2\pi\omega t); \quad \rho_1^0 = \frac{P_1 M_1}{R^* T_1}; \quad \rho_3^0 = \rho_1^0/(U_{10} + 1); \quad T_1 = 473 \text{ K};$$
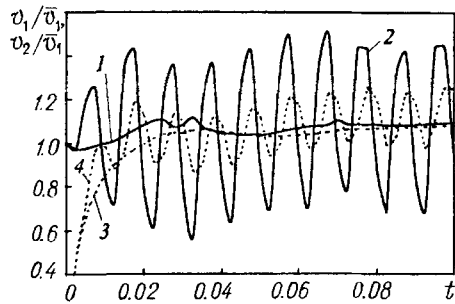
Fig. 1. Time dependence for the velocity of the gas (1, 2) and particles (3, 4) at the channel inlet ($X = 1$): 1, 3) $v_1^A = 0$; 2, 4) 30 m/sec.
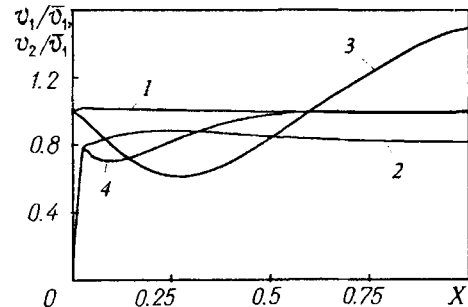
Fig. 2. Calculated curves for the velocity of the gas (1, 3) and particles (2, 4) in a channel: 1, 2) $v_1^A = 0$; 3, 4) 30 m/sec; $t = 0.01$ sec.
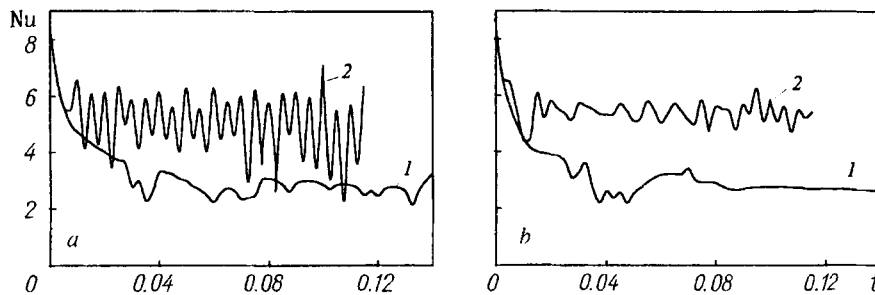
Fig. 3. Time dependence for the Nusselt number (a) $X = 1$; b) $X = 0.5$): 1) $v_1^A = 0$; 2) 30 m/sec.

$$\varepsilon_2 = 0.0005 \ ; \quad T_2 = 293 \ \text{K} \ ; \quad n = 6\varepsilon_2/\pi d^3 \ ; \quad v_2 = v_2^* \ ; \quad \rho_2^0 = 900 \ \text{kg/m}^3 \ .$$

Hence, the gas velocity at the channel inlet is a periodic function of time. The dispersed phase is also introduced here.

At the right boundary ($X = 1$) of the region, the flow was extrapolated beyond the isolated calculation domain.

The resulting system of equations was solved numerically using the McCormack explicit two-step difference scheme in conjunction with the method of flow correction [4, 5]. The calculation accuracy was controlled by a double recalculation with a half time step. An optimum time step was determined by the stability criteria and the assigned exactness of solution.

The calculations were performed for the following basic parameters: $d = 10^{-4}$ m, $U_{10} = 0.025$, $c_5 = 4190$ J/(kg·K), $c_3 = 1000$ J/(kg·K), $c_4 = 1850$ J/(kg·K), $\rho_6 = 500$ kg/m^3, $r = 2.26 \cdot 10^6$ J/kg, $M_1 = 29$ kg/kmole, $M_4 = 18$ kg/kmole, $R^* = 8314$ J/(kmole·K), $L = 4$ m, $\gamma = 1.4$, and $\omega = 100$ Hz.

Let us analyze the results of some variants of the calculation of the particle motion with account for the heat and mass transfer in the pulsating gas flow. At the channel inlet ($X = 0$), the velocity of the carrying phase (the gas) was taken as both a constant quantity and a periodic function of time $\bar{v}_1 + v_1^A \sin (2\pi\omega t)$. In the latter case, there is a wave motion of phases in the channel, which execute periodic vibrations with the same frequency but with a phase shift (Figs. 1 and 2). Here, the velocity amplitude for the gas is larger than for the particles. It should be noted that the time-average velocities of the gas and particles differ insignificantly, except for the section where particle acceleration occurs. In the case with no vibrations at the inlet, the gas velocity varies along the channel only a little, and its profile is close to a linear one. The phase velocities differ markedly only on the acceleration section, whose length for the considered particle diameter is small in comparison with the channel length. Therefore, the relative velocity of the phase motion $|v_1 - v_2|$ in this case is
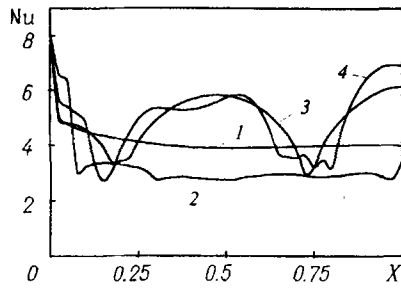
Fig. 4. Curve for the Nu number in a channel: 1, 2) $v_1^A = 0$; 3, 4) 30 m/sec (1, 3) $t = 0.02$ sec; 2, 4) 0.1 sec).
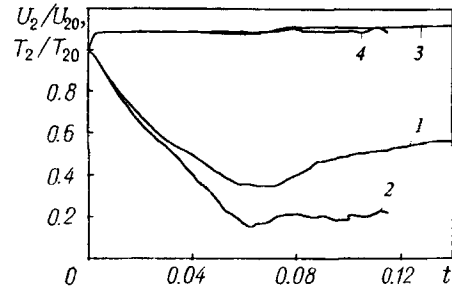
Fig. 5. Time dependence for the moisture content (1, 2) and temperature (3, 4) of particles at the channel outlet ($X = 1$): 1, 3) $v_1^A = 0$; 2, 4) 30 m/sec.
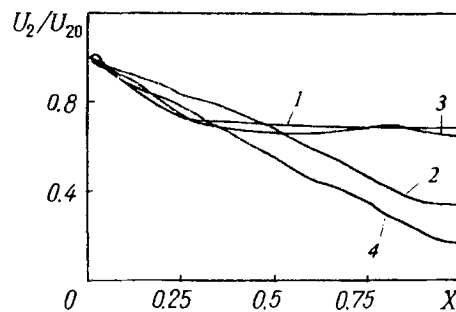

Fig. 6. Curves for the moisture content in a channel: 1, 2) $v_1^A = 0$; 3, 4) 30 m/sec [1, 3) $t = 0.02$ sec; 2, 4) 0.06 sec].

much smaller than in the presence of gas vibrations. As a result, the gas vibrations cause the coefficients of heat and mass transfer to increase.

The time dependence for the Nusselt number in the channel midsection and at the channel outlet is presented in Fig. 3, whence it is seen that, with the gas vibration, the time variation of the Nusselt number is of a wavy character. The mean value of Nu is noticeably larger than in the absence of vibrations. An insignificant difference is observed only on the acceleration section. It should be noted that, at the nodes of a standing wave, the Nu numbers in the two cases differ slightly, and in the loops their values with the gas vibration are maximum and much larger than in the absence of vibrations (Fig. 4). Thus, the gas vibrations enhance the heat and mass transfer. This is also supported by the time dependences of the moisture content (Fig. 5) and by the moisture-content profiles along the channel for instants of time of 0.02 and 0.06 sec (Fig. 6). It is seen from Fig. 6 that, in the initial period of time ($t = 0.02$ sec), the moisture contents of the particles in the two cases are slightly different. This results from the fact that, in this period, the particle acceleration occurs and the relative phase velocities are close. Subsequently, the vibrational mode leads to a higher-rate mass transfer and thus to a greater decrease in the moisture content of the particles. As the mode reaches a steady state, the moisture content varies insignificantly. In both cases, the particle temperature rises from the initial value to the temperature of adiabatic saturation of the gas (the wet-bulb temperature) and remains constant. Here, consideration is given to the first period of drying, i.e., to the removal of free moisture.

Thus, for the given formulation of the problem and the parameters considered it can be concluded that the specification of the gas vibrations at the boundary of the channel region in the form of a periodic function of time leads to an enhancement of the heat and mass transfer. In particular, this can be used in pneumatic tubes, which are employed for effecting the heat and mass transfer.

NOTATION

c, specific heat, J/(kg·K); d, particle diameter, m; D, diffusion coefficient, m^2/sec; E, total energy, J/kg; e, internal energy, J/kg; j_{12}, rate of phase change, kg/sec; L, channel length, m; M, molecular mass, kg/kmole; n, number of particles per unit volume; P, pressure, Pa; q, heat flux, W; r, heat of phase change, J/kg; R, particle radius, m; R^*, universal gas constant, J/(kmole·K); T, temperature, K; U, moisture content, kg/kg; v, velocity, m/sec; x, coordinate, m; $X = x/L$, dimensionless coordinate; γ, adiabatic exponent; ε, volume concentration of particles, m^3 of solid phase/m^3 of mixture; λ, thermal conductivity, W/(m·K); μ, dynamic viscosity, Pa·sec; ν, kinematic viscosity, m^2/sec; ρ_i, reduced phase density, which defines phase mass per unit volume of mixture (of solid phase and carrying phase), kg/m^3; ρ_i^0, at $n = 1, 2$, phase density, and at $n \neq 1, 2$, density of phase component, which defines component mass per unit volume of carrying phase, kg/m^3; t, time, sec; ω, frequency, Hz. Subscripts: 1, carrying phase; 2, dispersed phase; 3, dry air; 4, water vapor; 5, water; 6, dry particle; 0, initial (constant) value; s, saturation state. Superscripts: 0, true values; A, amplitude.

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